AD-A191 892 CONVERGENCE RATE OF CODES FOR NUMERICAL QUARDRATURE TECHNIQUES FOR CLASSICAL RAY TRACING(U) NAVAL OCEAN SYSTEMS CENTER SAN DIEGO CA E R FLOYD DEC 86 1/1

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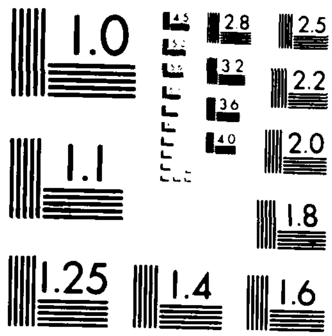
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The estimated residual error (or its bound) for numerical quadratures is usually expressed in terms of a derivative of some order of the integrand or some residual factor of the integrand after factoring out a countable number of zeroes and singularities that occur along the integration path. The order of the derivative is a function of the number of sample points for evaluating the integrand. Regrettably, the magnitudes of these higher order derivatives are difficult enough to estimate for even analytic sound velocity profiles. In practice, observed sound velocity profiles, which are usually given in tabular form and include measurement errors, exacerbate our inability to assess the magnitudes of these higher order derivatives. An estimate of the residual error expressed in terms of a first derivative would be far more practical for both analytic and observed sound velocity profiles.

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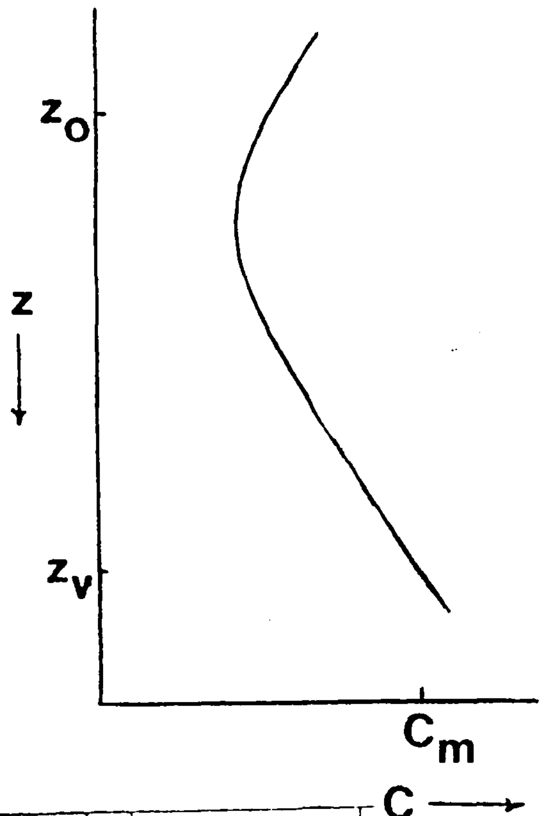
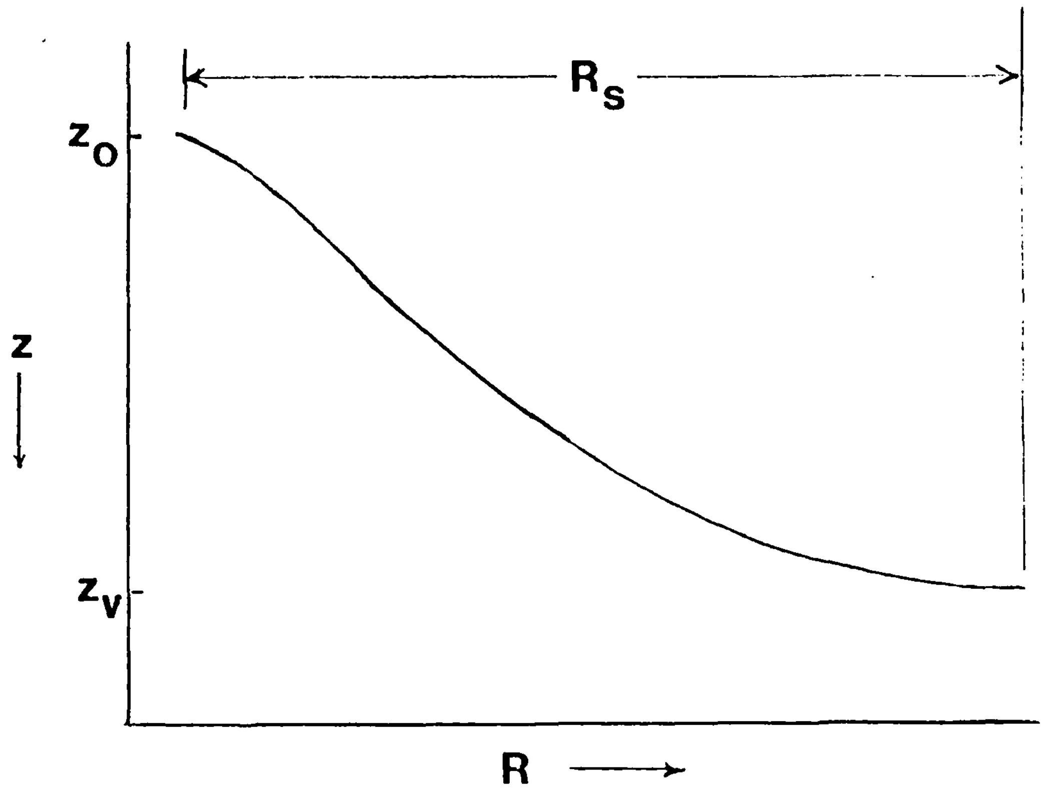
CONVERGENCE RATE OF CODES FOR NUMERICAL QUADRATURE TECHNIQUES FOR CLASSICAL RAY TRACING

Edward R. Floyd

Arctic Submarine Laboratory

By classical ray tracing, determine range over a single Riemann sheet for a ray described by the constant of motion (vertex velocity) C_m. How accurate?





Range Integral:

$$R_s = \int_{z_0}^{z_v} \frac{C(z)}{[C_m^2 - C^2(z)]^{1/2}}\, dz$$

$$= \int_{z_0}^{z_v} W(z)\, f(z)\, dz$$

W and f are convenient functions of z. [W dz] becomes a Stieltjes measure and f(z), the Stieltjes integrand.

Trick: Choose W(z) to incorporate any poor behavior (singularities or zeros) into W leaving f(z) well behaved.

Thus the Gaussian quadrature

$$R_s \simeq (z_v - z_o)^m \sum_{i=1}^{n} w_{i,n}\, f(z_{i,n})$$

where:

n is the order of the approximation;

m is determined by W;

$w_{i,n}$ is a Gaussian weighting coefficient determined by W;

$z_{i,n}$ is a Gaussian sample point determined by W.

The Gaussian error is given by

$$\eta_n = \frac{f^{(2n)}(\xi)}{(2n)!} \int_{z_0}^{z_v} W(z) \left[\prod_{i=1}^{n} (z-z_{i,n})\right]^2 dz$$

where $z_0 \leq \xi \leq z_v$.

For underwater acoustics

$$W = (z_v - z)^{-1/2}.$$

Factors out branch point singularity at vertex depth.

Therefore

$$\eta_n = \frac{\pi^{1/2} (z_v - z_0)^{2n+1}}{4n+1} \frac{[(2n)!]^3}{[(4n)!]^2} f^{(2n)}(\xi),$$

$$z_0 \leq \xi \leq z_v.$$

$$\eta_n = \frac{\pi^{1/2} (z_v - z_0)^{2n+1}}{4n+1} \frac{[(2n)!]^3}{[(4n)!]^2} f^{(2n)}(\xi),$$

$$z_0 \leq \xi \leq z_v.$$

This expression has several deficiencies;

- Usually impractical to evaluate $f^{(2n)}$.
- $f^{(i)}$ for $i < 2n$ may be discontinuous.
- $C(z)$ is often known only in tabular form.

$$\eta_n = \frac{\pi^{1/2} (z_V - z_O)^{2n+1}}{4n+1} \frac{[(2n)!]^3}{[(4n)!]^2} f^{(2n)}(\xi),$$

$$z_O \leq \xi \leq z_V.$$

Analytic (hypothetical) C(z):*

- Rapid convergence with increasing n.
- Only limited by round-off error.
- Thumb rule for maximum effective n:

  $n_{max} \approx$ ("number of signifcant figures")/2

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* E. R. Floyd, J. Acoust. Soc. Am. 49, 1580-1590 (1971)

$$\eta_n = \frac{\pi^{1/2}\,(z_v - z_0)^{2n+1}}{4n+1} \; \frac{[(2n)!]^3}{[(4n)!]^2} \; f^{(2n)}(\xi),$$

$$z_0 \leq \xi \leq z_v.$$

Observed (nonanalytic) C(z):*

- C(z) tabled at discrete depths.
- Cubic spline fit $\longrightarrow$ d^3C/dz^3 discontinuous. Nevertheless still rapidly convergent over an assembly of 116 observed C(z)'s. MOE --- η reduced by a factor of 0.763 for each additional sample point per Riemann sheet.
- Thumb rule for maximum effective n:

$$n_{max} \geq \text{"number of significant figures".}$$

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* E. R. Floyd and J. D. Pugh, J. Acoust. Soc. Am. 61, 682-687 (1977).

Present Status:

- Even for analytic functions, quadrature errors are difficult to estimate because higher order derivitives are generally complicated.

- While we have convergence rates for an assembly of observed profiles, the discontinuity of higher order derivatives confounds our estimate of convergence for a particular observed profile.

→ Desire an estimate expressed in terms of the first derivative of the Stieltjes integrand.

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